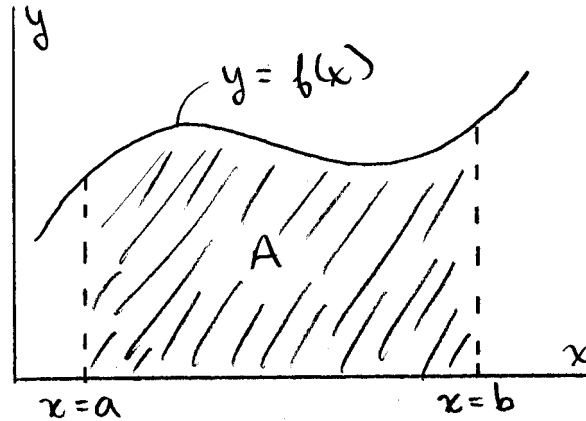


# The Integral

The definite integral  $\int_a^b f(x)dx$  is the area under the function  $y = f(x)$  between points  $a$  and  $b$ :



$$A = \int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a),$$

where  $F(x) = \int f(x)dx$  is the *antiderivative* of  $f(x)$ .

Antiderivatives of some common functions in engineering:

Function, $f(x)$	Antiderivative, $F(x) = \int f(x)dx$
$\sin(\omega x)$	$-\frac{1}{\omega} \cos(\omega x) + C$
$\cos(\omega x)$	$\frac{1}{\omega} \sin(\omega x) + C$
$e^{sx}$	$\frac{1}{s} e^{sx} + C$
$x^n$	$\frac{x^{n+1}}{n+1} + C$
$cf(x)$	$c \int f(x)dx$
$f_1(x) + f_2(x)$	$\int f_1(x)dx + \int f_2(x)dx$

- In the above table,  $\omega$ ,  $s$ ,  $n$ ,  $c$  and  $C$  are constants (not functions of  $x$ )



**TABLE OF INTEGRALS** . . . . .

**BASIC FORMS**

- |  |   |
|--|---|
| 1. $\int u \, dv = uv - \int v \, du$                          | 11. $\int \csc u \cot u \, du = -\csc u + C$  |
| 2. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$ | 12. $\int \tan u \, du = \ln  \sec u  + C$  |
| 3. $\int \frac{du}{u} = \ln  u  + C$                           | 13. $\int \cot u \, du = \ln  \sin u  + C$  |
| 4. $\int e^u \, du = e^u + C$                                  | 14. $\int \sec u \, du = \ln  \sec u + \tan u  + C$                                   |
| 5. $\int a^u \, du = \frac{a^u}{\ln a} + C$                    | 15. $\int \csc u \, du = \ln  \csc u - \cot u  + C$                                   |
| 6. $\int \sin u \, du = -\cos u + C$                           | 16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$                    |
| 7. $\int \cos u \, du = \sin u + C$                            | 17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$               |
| 8. $\int \sec^2 u \, du = \tan u + C$                          | 18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$       |
| 9. $\int \csc^2 u \, du = -\cot u + C$                         | 19. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left  \frac{u+a}{u-a} \right  + C$ |
| 10. $\int \sec u \tan u \, du = \sec u + C$                    | 20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left  \frac{u-a}{u+a} \right  + C$ |

**FORMS INVOLVING  $\sqrt{a^2 + u^2}, a > 0$**

21.  $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
22.  $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$
23.  $\int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$
24.  $\int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$
25.  $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$
26.  $\int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
27.  $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
28.  $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29.  $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$

