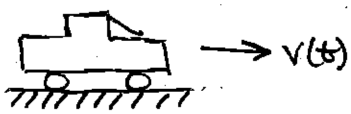
Straight Lines in Engineering

Example: (ME2210 – Dynamics)

During braking, the velocity of a vehicle satisfies the relation

$$v(t) = v_0 + at$$



v_0 ... initial velocity (m/s)
a ... acceleration (m/s²)

Determine the initial velocity v_0 and acceleration a if the velocity is known at the following two points. Also find the ~~total~~ stopping time.

t (s)	v(t) (m/s)
0.75	35
1.25	2.4

Solution: the equation is in the form $y = mx + b$, where $v = y$, $m = a$, $x = t$, $b = v_0$

→ slope, $m = a = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{v_2 - v_1}{t_2 - t_1}$

$$\therefore a = \frac{2.4 - 35}{1.25 - 0.75} = -65.2 \qquad \boxed{\therefore a = -65.2 \text{ m/s}^2}$$

→ intercept, $b = v_0$, determined using slope-intercept form and either data point

$$y = mx + b \Rightarrow v = -65.2t + v_0$$

point 1: $(t, v) = (0.75, 35)$

$$35 = -65.2(0.75) + v_0$$

$$v_0 = 35 + 65.2(0.75) = 83.9 \qquad \boxed{\therefore v_0 = 83.9 \text{ m/s}}$$

same result from using point 2: $(t, v) = (1.25, 2.4)$

$$2.4 = -65.2(1.25) + v_0$$

$$v_0 = 2.4 + 65.2(1.25) = \underline{83.9} \checkmark$$

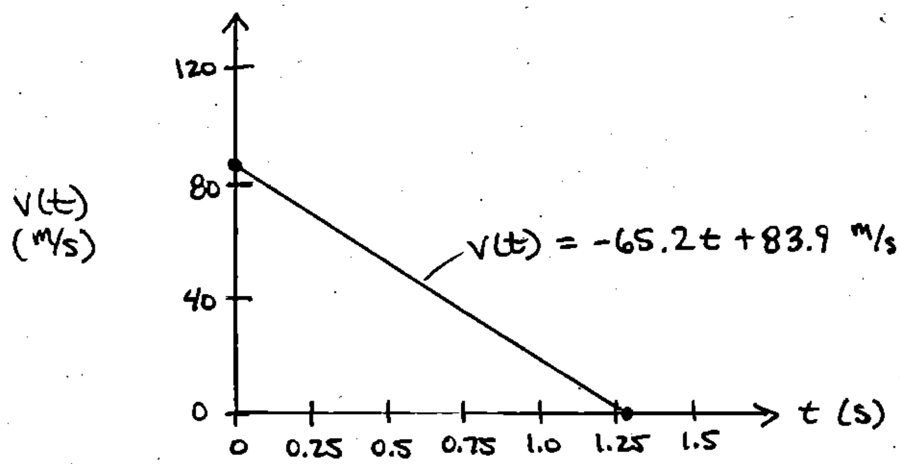
→ total stopping time (time required to reach $V = 0$)

$$v(t) = V_o + at = 83.9 + -65.2t$$

$$V = 0 = 83.9 - 65.2t \qquad \text{(solve for } t\text{)}$$

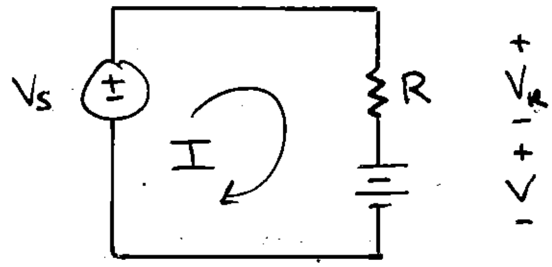
$$\Rightarrow 65.2t = 83.9 \qquad \therefore t = 1.29\,s$$

<u>NOTE</u>: The stopping time $t = 1.29\,s$ and initial velocity $V_o = 83.9\,\frac{m}{s}$ are the x & y intercepts of the line



Example: (EE 2010 – Circuits)

For the electric circuit,



$V_R = IR$ (Ohm's Law)

V_s... applied voltage source (volts, V)
I... current (amps, A)
R... resistance (ohms, Ω)
V... unknown voltage source

Governing equation for the circuit:

Kirchoff's Voltage Law (KVL),

$$\Sigma \text{ voltage rises} = \Sigma \text{ voltage drops}$$

$$\Rightarrow \quad V_s = V_R + V$$

or $\boxed{V_s = IR + V}$

Above is the equation of a line relating the voltage source V_s to the current I.

Find the values of R and V if the values of V_s and I are known as the following two points.

V_s (volts)	I (amps)
10	0.1
20	1.1

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Solution: $V_s = IR + V$

$\Rightarrow$ $V_s = R\,I + V$

$y = m\,x + b$

$\rightarrow$ slope, $m = \frac{\Delta y}{\Delta x} = \frac{\Delta V_s}{\Delta I} = \frac{20-10}{1.1-0.1} = 10$ $\quad \therefore m = 10.$

Slope-intercept form: $V_s = 10I + b$

using $(I, V_s) = (0.1, 10)$ $\Rightarrow$ $10 = 10(0.1) + b$ $\quad \therefore b = 9$

$\Rightarrow V_s = 10I + 9$

point-slope form: $(y - y_1) = m(x - x_1)$

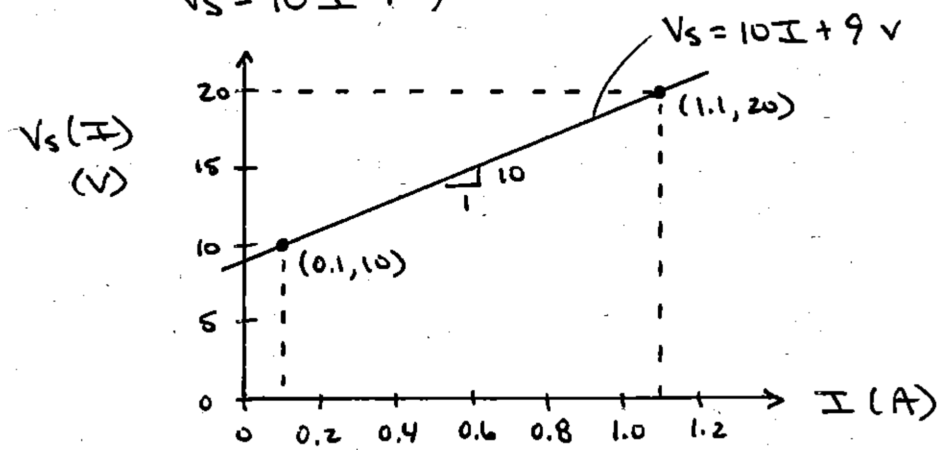
$V_s - 10 = 10(I - 0.1) \Rightarrow V_s = 10I - 1.0 + 10$

$\boxed{\therefore V_s = 10I + 9 \text{ volts}}$

Since $V_s = RI + V$, $\boxed{R = 10\,\Omega \;\; \& \;\; V = 9 \text{ volts}}$

Graph of V_s vs. I: (y vs. x)

$V_s = 10I + 9$



Alternative Solution: $V_s = IR + V$

treat V_s as the independent variable

$$\Rightarrow \quad I = \frac{1}{R} V_s - \frac{V}{R}$$

$$(y = m x + b)$$

$\rightarrow$ slope, $m = \frac{\Delta y}{\Delta x} = \frac{\Delta I}{\Delta V_s} = \frac{1.1 - 0.1}{20 - 10} = 0.1 \qquad \therefore m = 0.1$

$\rightarrow$ intercept, $I = 0.1\, V_s + b$

using $(V_s, I) = (10, 0.1)$

$$0.1 = 0.1(10) + b \quad \Rightarrow \quad b = -0.9$$

or $\boxed{I = 0.1\, V_s - 0.9}$

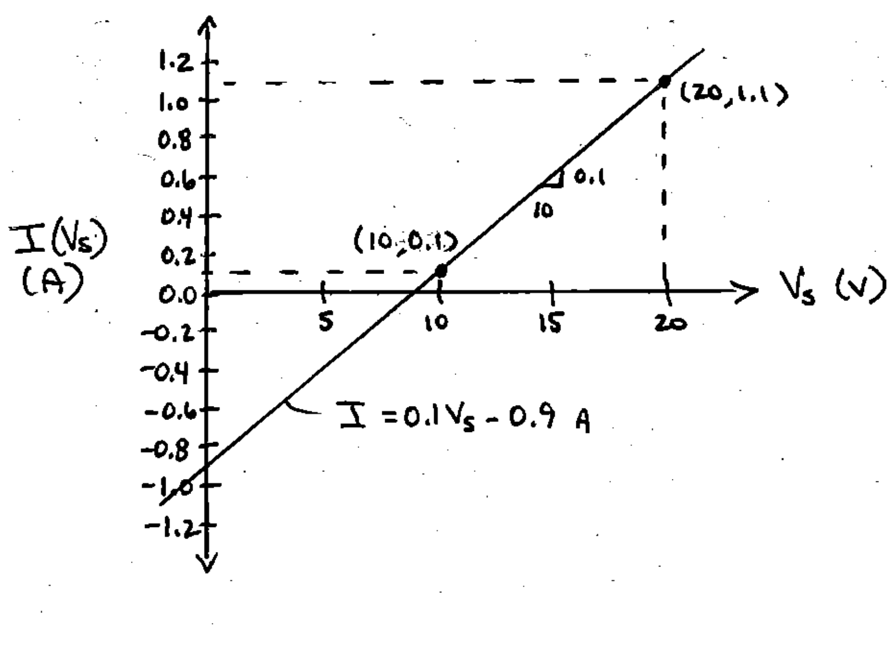
since $I = \frac{1}{R} V_s - \frac{V}{R}$

$$\Rightarrow \frac{1}{R} = 0.1 \qquad \therefore R = 10\,\Omega$$

$$\Rightarrow -\frac{V}{R} = -0.9 \Rightarrow V = 0.9R = 0.9(10) = 9 \qquad \therefore V = 9\,v$$

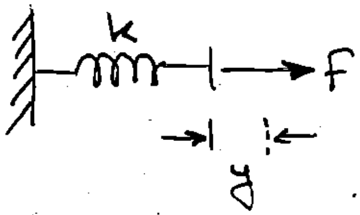
NOTE: we get same answers as before! $\boxed{R = 10\,\Omega \;;\; V = 9 \text{ volts}}$

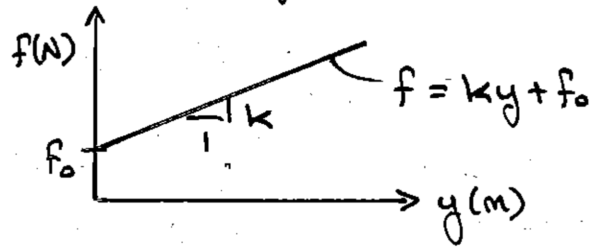
Graph of I vs. V_s:



Example: (PHY 2400 - Physics)

Force - Deflection in a pre-loaded spring,





$$f = ky + f_0$$

f ... force (N)

k ... spring constant (N/m)

y ... displacement (m)

f_0 ... pre-load (N)

Find the spring rate k and the preload f_0 if the following values of force and displacement are known for the system:

f (N)	y(m)
1.0	0.1
5.0	0.9

Solution: method 1 - treat y as the independent variable

$$f = ky + f_0$$
$$(y = mx + b)$$

→ slope, $m = \frac{\Delta y}{\Delta x} = \frac{\Delta f}{\Delta y} = \frac{5-1}{0.9-0.1} = \frac{4}{0.8} = 5$ $\quad \therefore m = 5$

→ intercept, $f = 5y + b$

using $(0.9, 5)$ $\Rightarrow$ $5 = 5(0.9) + b$ $\quad \therefore b = 0.5$

$\boxed{f = 5y + 0.5 \text{ N}}$ since $f = ky + f_0$, $\boxed{k = 5 \frac{N}{m} \; ; \; f_0 = 0.5 \text{ N}}$

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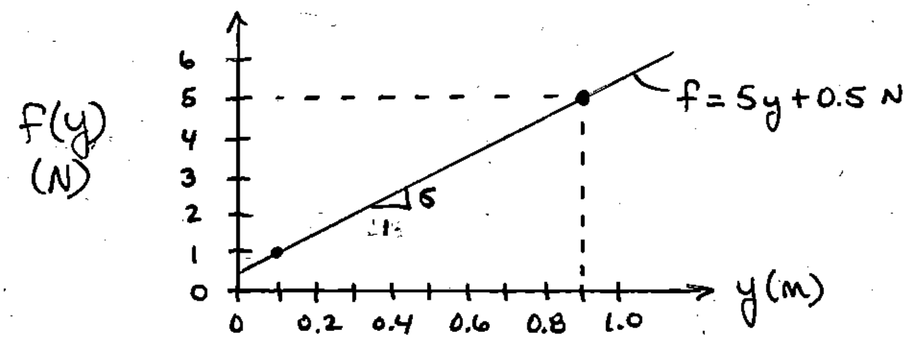
Graph of f vs. y:



Alternative Solution: method 2 - treat f as the independent variable

$$f = ky + f_0 \qquad \text{(solve for } y\text{)}$$

$$y = \frac{1}{k} f - \frac{f_0}{k}$$

$$(y = m x + b)$$

→ slope, $m = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta f} = \frac{0.8}{4} = 0.2 \qquad \therefore m = 0.2$

→ intercept, $y = 0.2f + b$

using $(5, 0.9)$, $0.9 = 0.2(5) + b \qquad \therefore b = -0.1$

then $\boxed{y = 0.2f - 0.1 \text{ m}}$

since $y = \frac{1}{k} f - \frac{f_0}{k}$, $\quad \frac{1}{k} = 0.2 \quad \Rightarrow k = 5 \text{ N/m}$

$-\frac{f_0}{k} = -0.1 \quad \Rightarrow f_0 = 0.5 \text{ N}$

Graph of y vs. f,