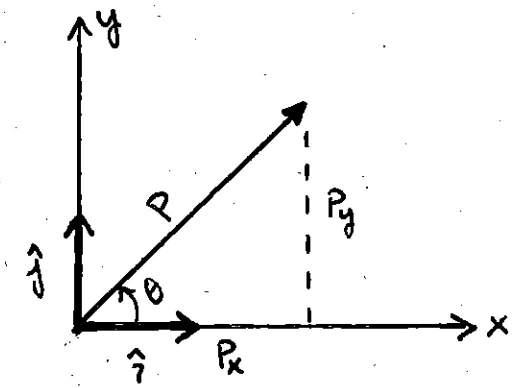
Two-Dimensional Vectors in Engineering

The position of the tip of a one-link robot can be represented as a 2-D vector, $\vec{P}$:



<u>Position Vector</u>: $\vec{P} = P_x\hat{i} + P_y\hat{j}$

$\hat{i}$ ... unit vector in x-direction (sometimes noted as $\vec{i}$)

$\hat{j}$ ... unit vector in y-direction (sometimes noted as $\vec{j}$)

P_x ... x-component, $P_x = P\cos\theta$

P_y ... y-component, $P_y = P\sin\theta$

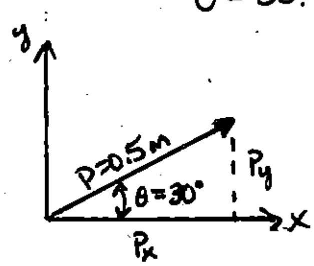
$P = \sqrt{(P_x)^2 + (P_y)^2}$ ... <u>magnitude</u> of $\vec{P}$

$\theta = \text{atan}2(P_y/P_x)$ ... <u>direction</u> of $\vec{P}$

(Actual MATLAB syntax is atan2(P_y, P_x))

Here a vector is an engineering quantity that has both <u>magnitude</u> and <u>direction</u>.

<u>EXAMPLE</u>: The magnitude is $P = 0.5\,m$ and the direction is $\theta = 30°$. Find P_x and P_y and write $\vec{P}$ in vector notation.



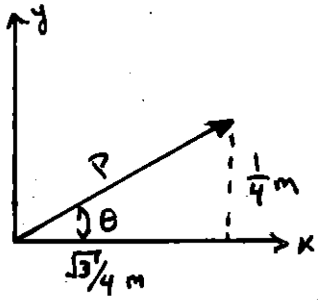
$P_x = 0.5\cos 30 = 0.5\,\sqrt{3}/2 = 0.433$

$P_y = 0.5\sin 30 = 0.5\,(1/2) = 0.25$

$$\boxed{\vec{P} = 0.433\hat{i} + 0.25\hat{j}\ m}$$

TOPS 35500

<u>EXAMPLE</u>: Given $P_x = \sqrt{3}/4$ and $P_y = 1/4$, find the magnitude and direction of $\vec{P}$.



$$P = \sqrt{(P_x)^2 + (P_y)^2} = \sqrt{(\sqrt{3}/4)^2 + (\tfrac{1}{4})^2}$$

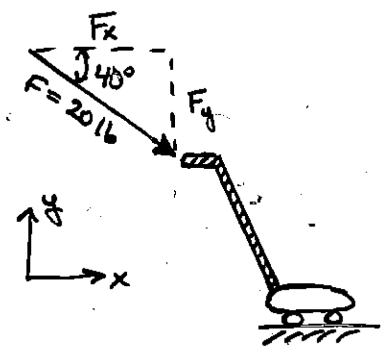
$$\Rightarrow P = 0.5 \text{ m}$$

$$\theta = \text{atan2}\left(\frac{1/4}{\sqrt{3}/4}\right) = \text{atan2}\left(1/\sqrt{3}\right) = 30^\circ$$

$$\therefore \vec{P} = 0.5 \text{ m @ } 30^\circ \rightarrow \boxed{\vec{P} = 0.5 \angle 30^\circ \text{ m} \quad \text{"polar form"}}$$

<u>EXAMPLE</u>: (ME2120 – Statics)

A person pushes a vacuum cleaner with a force of F = 20 lb at an angle of 40° relative to the ground. Determine the horizontal and vertical components of the force.



$$F_x = F\cos 40^\circ = 20\cos 40^\circ = 15.32 \text{ lb}$$

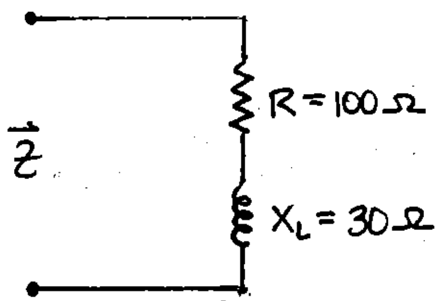
$$F_y = -F\sin 40 = -20\sin 40^\circ = -12.86 \text{ lb}$$

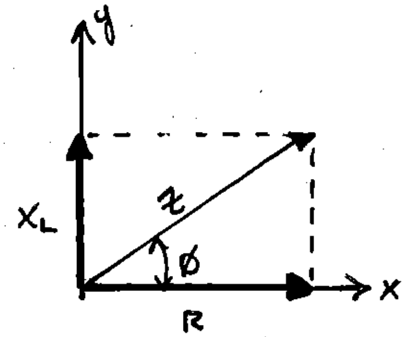
$$\Rightarrow \vec{F} = 15.32\hat{\imath} - 12.86\hat{\jmath} \text{ lb}$$

(<u>NOTE</u>: negative indicates $-y$ direction)

EXAMPLE: Impedance of Inductor and Resistor in Series

(EE 2010 - Circuits)





X_L ... impedance of inductor (Ω)

R ... impedance of resistor (resistance, Ω)

$z = \sqrt{R^2 + X_L^2}$ ... total impedance

$\phi = \text{atan}2\left(\frac{X_L}{R}\right)$ ... phase angle

Vector Notation: $\vec{z} = R\,\hat{\imath} + X_L\,\hat{\jmath}\ \Omega$

Here, $R = 100\,\Omega$, $X_L = 30\,\Omega$

$$\Rightarrow \vec{z} = 100\,\hat{\imath} + 30\,\hat{\jmath}\ \Omega$$

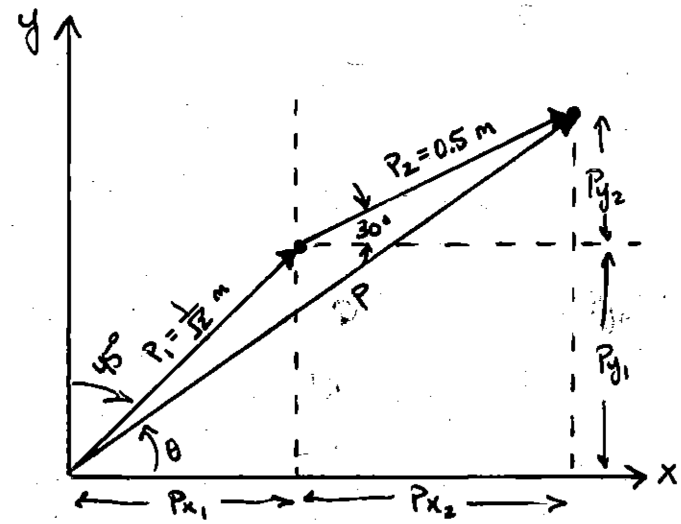
Total Impedance: $z = \sqrt{(100^2) + (30^2)} = 104.4\,\Omega$

Phase Angle: $\phi = \text{atan}2(30/100) \doteq 16.7°$

$$z = 104.4 \angle 16.7°\ \Omega$$

Vector Addition – 2 Link Robot:

The position of the tip of the two link robot can be written in vector form through vector addition of the position of each link:



For the above configuration, determine P and θ by vector addition,

Solution: $\vec{P} = \vec{P}_1 + \vec{P}_2 = P_x \hat{\imath} + P_y \hat{\jmath}$

where $\vec{P}_1 = P_{x_1} \hat{\imath} + P_{y_1} \hat{\jmath}$

and $\vec{P}_2 = P_{x_2} \hat{\imath} + P_{y_2} \hat{\jmath}$

$$\Rightarrow \quad \vec{P} = (P_{x_1} \hat{\imath} + P_{y_1} \hat{\jmath}) + (P_{x_2} \hat{\imath} + P_{y_2} \hat{\jmath})$$

$$\vec{P} = (P_{x_1} + P_{x_2}) \hat{\imath} + (P_{y_1} + P_{y_2}) \hat{\jmath}$$

since $\vec{P} = P_x \hat{\imath} + P_y \hat{\jmath}$

$$P_x = P_{x_1} + P_{x_2} \quad \text{and} \quad P_y = P_{y_1} + P_{y_2}$$

$\Rightarrow$ adding 2 vectors amounts to adding their x & y components!

Here, $P_{x_1} = P_1 \cos 45° = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = 0.5 \text{ m}$

$P_{y_1} = P_1 \sin 45° = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = 0.5 \text{ m}$

$$\Rightarrow P_1 = 0.5\,\hat{\imath} + 0.5\,\hat{\jmath} \text{ m}$$

Similarly, $P_{x_2} = P_2 \cos 30° = (0.5)(\sqrt{3}/2) = 0.433 \text{ m}$

$P_{y_2} = P_2 \sin 30 = (0.5)(\tfrac{1}{2}) = 0.250 \text{ m}$

$$\Rightarrow P_2 = 0.433\,\hat{\imath} + 0.250\,\hat{\jmath}$$

Finally, $\vec{P} = \vec{P}_1 + \vec{P}_2$

$$\vec{P} = (0.5\,\hat{\imath} + 0.5\,\hat{\jmath}) + (0.433\,\hat{\imath} + 0.250\,\hat{\jmath})$$

$$\vec{P} = 0.933\,\hat{\imath} + 0.750\,\hat{\jmath}$$

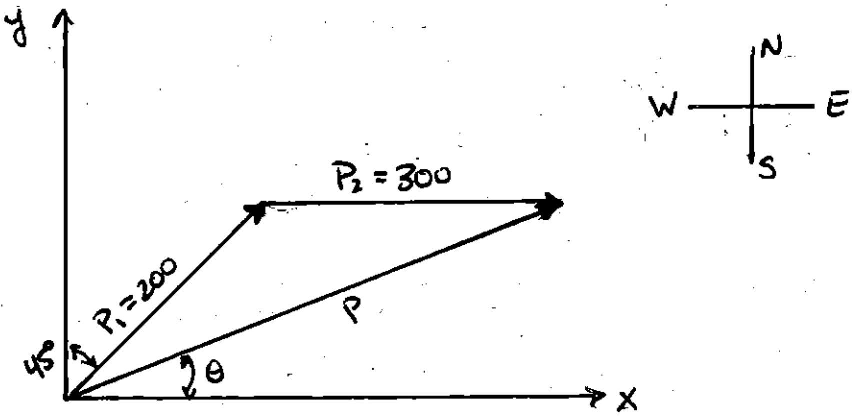
$$P = \sqrt{(0.933)^2 + (0.750)^2} = 1.197 \text{ m}$$

$$\theta = \text{atan2}(0.75/0.933) = 38.79°$$

$$\boxed{P = 1.197 \angle 38.79°}$$

Example :

A ship travels 200 miles @ 45° North of East, then 300 miles due East. Find the resulting position of the ship.



$$\vec{P} = \vec{P}_1 + \vec{P}_2$$

$$\vec{P}_1 = 200\cos 45^\circ\,\hat{i} + 200\sin 45^\circ\,\hat{j}$$

$$\vec{P}_1 = 141.4\,\hat{i} + 141.4\,\hat{j} \text{ miles}$$

$$\vec{P}_2 = 300\,\hat{i} + 0\,\hat{j} \text{ miles}$$

$$\vec{P} = (141.4\,\hat{i} + 141.4\,\hat{j}) + (300\,\hat{i})$$

$$\boxed{\therefore \vec{P} = 441.4\,\hat{i} + 141.4\,\hat{j} \text{ miles}}$$

magnitude and direction:

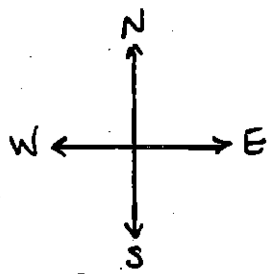
$$P = \sqrt{(441.4)^2 + (141.4)^2} = 463.5 \text{ miles}$$

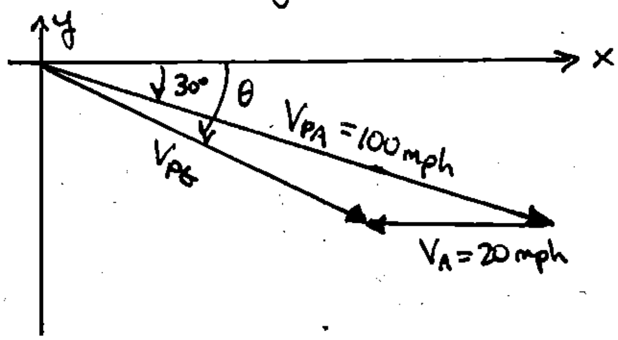
$$\theta = \text{atan2}(141.4/441.4) = 17.76^\circ$$

$$\boxed{\vec{P} = 463.5 \measuredangle 17.76^\circ \text{ miles}}$$

<u>Example</u>: Relative Velocity (ME 2210 - Dynamics)

A airplane flies at an airspeed of 100 mph @ a heading of 30° south of east. If the velocity of the wind is 20 mph due west, determine the resultant velocity of the plane with respect to the ground.





$$\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_A$$

$\vec{V}_{PG}$ ... velocity of plane relative to ground (mph)

$\vec{V}_{PA}$ ... velocity of plane relative to air (airspeed) (mph)

$\vec{V}_A$ ... velocity of the wind relative to ground (mph)

Here, $\vec{V}_{PA} = 100\cos 30°\,\hat{\imath} - 100\sin 30°\,\hat{\jmath}$

$\vec{V}_{PA} = 50\sqrt{3}\,\hat{\imath} - 50\,\hat{\jmath}$ mph

Also, $\vec{V}_A = -20\,\hat{\imath} + 0\,\hat{\jmath}$ mph

$\Rightarrow \quad \vec{V}_{PG} = (50\sqrt{3}\,\hat{\imath} - 50\,\hat{\jmath}) + (-20\,\hat{\imath} + 0\,\hat{\jmath})$

$\vec{V}_{PG} = (50\sqrt{3} - 20)\,\hat{\imath} - 50\,\hat{\jmath}$

$$\boxed{\vec{V}_{PG} = 66.6\,\hat{\imath} - 50\,\hat{\jmath} \text{ mph}}$$

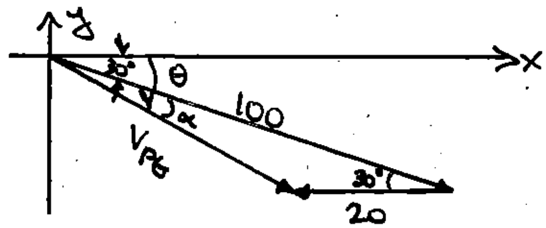
magnitude and direction:

$$V_{PG} = \sqrt{(66.6)^2 + (-50)^2} = 83.3 \text{ mph}$$

$$\theta = \text{atan2}\left(\frac{-50}{66.6}\right) = -36.9°$$

$$\boxed{V_{PG} = 83.3 \angle -36.9° \text{ mph}}$$

Aside: Can also solve via Laws of Cosines & Sines



$$(V_{PG})^2 = (20)^2 + (100)^2 - 2(20)(100)\cos 30°$$

$$(V_{PG})^2 = 6936 \Rightarrow V_{PG} = 83.28 \text{ mph } \checkmark$$

also, $$\frac{\sin\alpha}{20} = \frac{\sin 30°}{V_{PG}} = \frac{1/2}{83.28}$$

$$\Rightarrow \alpha = 6.896°$$

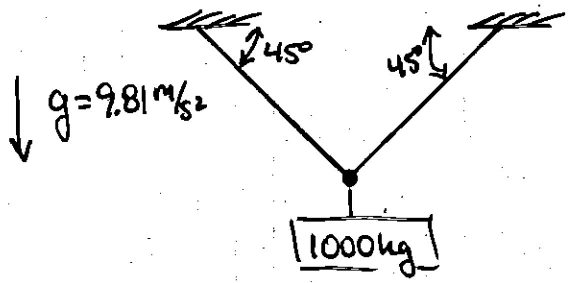
$$\Rightarrow \theta = 30 + \alpha = 36.89° \checkmark$$

again, $\vec{V}_{PG} = 83.3 \angle -36.9° \text{ mph}$

TOPS 35500

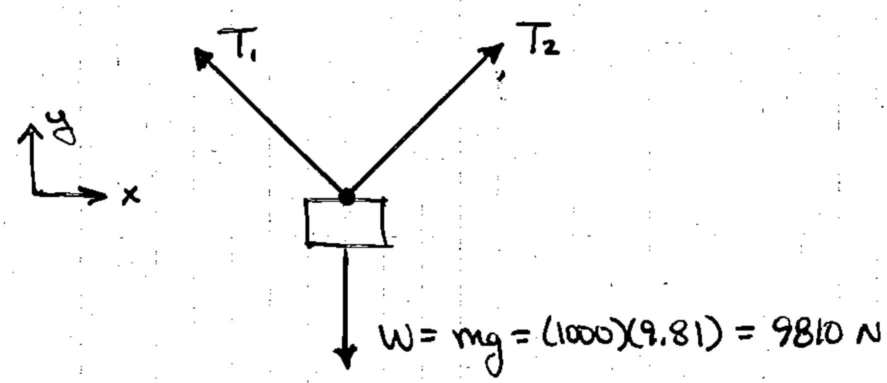
EXAMPLE: Static Equilibrium (ME 2120 - Statics)

A 1000 kg object hangs from two cables of equal length:



Determine the tension in each cable.

Free-body diagram (FBD):



Equilibrium: $\Sigma \vec{F} = 0$

$$\vec{T}_1 + \vec{T}_2 + \vec{W} = 0$$

Here,
$$\vec{T}_1 = -T_1 \cos 45° \hat{\imath} + T_1 \sin 45° \hat{\jmath} \text{ N}$$
$$\vec{T}_2 = T_2 \cos 45° \hat{\imath} + T_2 \sin 45° \hat{\jmath} \text{ N}$$
$$\vec{W} = 0 \hat{\imath} + (-9810)\hat{\jmath} = -9810 \hat{\jmath} \text{ N}$$

$$\Rightarrow \quad \left(-\frac{T_1}{\sqrt{2}} \hat{\imath} + \frac{T_1}{\sqrt{2}} \hat{\jmath}\right) + \left(\frac{T_2}{\sqrt{2}} \hat{\imath} + \frac{T_2}{\sqrt{2}} \hat{\jmath}\right) + \left(-9810 \hat{\jmath}\right) = 0$$

$$\Rightarrow \quad \left(-\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{2}}\right) \hat{\imath} + \left(\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{2}} - 9810\right) \hat{\jmath} = 0 \hat{\imath} + 0 \hat{\jmath}$$

Equating $\hat{i}$ & $\hat{j}$ components to zero, gives 2 equations to solve for 2 unknowns T_1 & T_2

$$\Sigma F_x \Rightarrow \quad \left(-\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{2}}\right) = 0 \quad \Rightarrow \quad T_1 = T_2 \quad \ldots (1)$$

$$\Sigma F_y \Rightarrow \quad \left(\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{2}} - 9810\right) = 0 \quad \ldots (2)$$

Subbing (1) into (2),

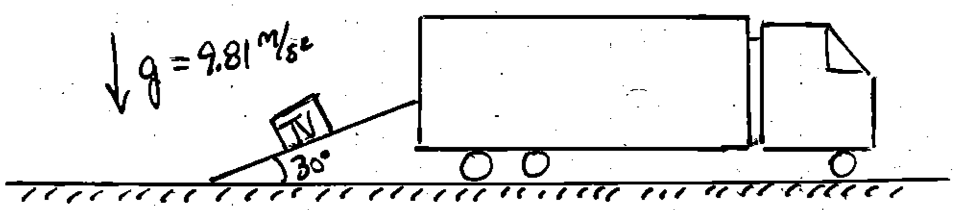
$$\frac{T_1}{\sqrt{2}} + \frac{T_1}{\sqrt{2}} - 9810 = 0$$

$$\frac{2T_1}{\sqrt{2}} = 9810 \quad \Rightarrow \quad T_1 = \left(\frac{\sqrt{2}}{2}\right)(9810) = 6937 \text{ N}$$

$$\boxed{T_1 = T_2 = 6937 \text{ N}}$$

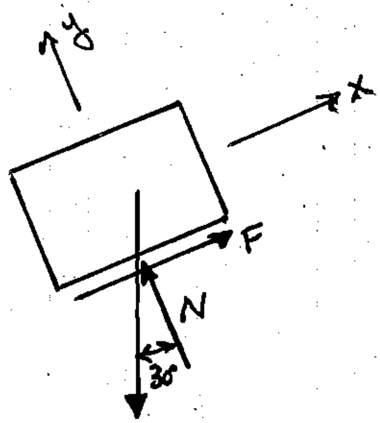
Example: Static Equilibrium (ME 2120 - Statics)

A 100 kg TV is loaded onto a truck using a ramp at a 30° angle. Find the normal and frictional forces on the TV if it is left sitting on the ramp.



Free-body Diagram (FBD) for TV,

(Note rotated coordinate axes)



$W = mg = (100)(9.81) = 981\ N$

Solution:

$\Sigma \vec{F} = 0$

$\vec{F} + \vec{N} + \vec{W} = 0$

Here, $\vec{F} = F\hat{i}\ \ N$

$\vec{N} = N\hat{j}\ \ N$

$\vec{W} = -981 \sin 30\,\hat{i} - 981 \cos 30\,\hat{j}$

$\vec{W} = -490.5\,\hat{i} - 849.6\,\hat{j}\ \ N$

$\vec{F} + \vec{N} + \vec{W} = 0$

$F\hat{i} + N\hat{j} - 490.5\,\hat{i} - 849.6\,\hat{j} = 0$

Equating $\hat{i}$ & $\hat{j}$ components to zero,

$F - 490.5 = 0 \quad \ldots (1)$

$N - 849.6 = 0 \quad \ldots (2)$

$$\therefore \boxed{\begin{aligned} F &= 490.5\ N \\ N &= 849.6\ N \end{aligned}}$$