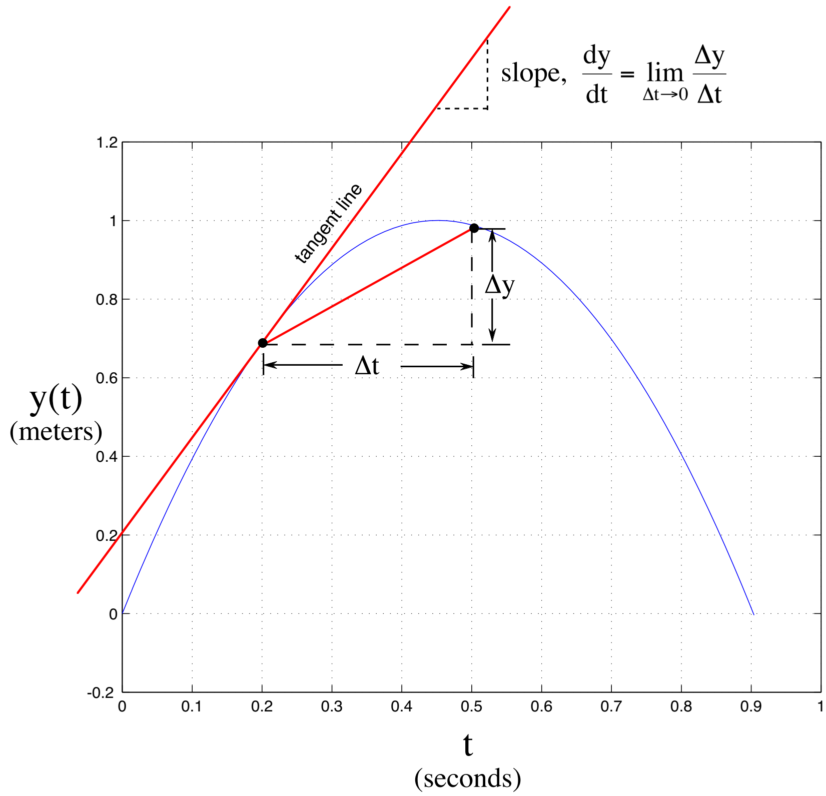
The Derivative

The derivative *dy/dt* of a function *y(t)* is the slope of the tangent line to that function at time *t*:



Derivatives of some common functions in engineering:

Function, *y(t)*	Derivative, *dy/dt*
$sin(\omega t)$	$\omega cos(\omega t)$
$cos(\omega t)$	$-\omega sin(\omega t)$
e^{st}	se^{st}
t^n	nt^{n-1}
$cy(t)$	cdy/dt
$y_1(t)+y_2(t)$	$dy_1/dt + dy_2/dt$

In the above table, *ω*, *s*, *n* and *c* are constants (not functions of *t*).

DIFFERENTIATION RULES

GENERAL FORMULAS

1. $\frac{d}{dx}(c) = 0$

2. $\frac{d}{dx}[cf(x)] = cf'(x)$

3. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

4. $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

5. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule)

6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Quotient Rule)

7. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)

8. $\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

9. $\frac{d}{dx}(e^x) = e^x$

10. $\frac{d}{dx}(a^x) = a^x \ln a$

11. $\frac{d}{dx}\ln|x| = \frac{1}{x}$

12. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

TRIGONOMETRIC FUNCTIONS

13. $\frac{d}{dx}(\sin x) = \cos x$

14. $\frac{d}{dx}(\cos x) = -\sin x$

15. $\frac{d}{dx}(\tan x) = \sec^2 x$

16. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

17. $\frac{d}{dx}(\sec x) = \sec x \tan x$

18. $\frac{d}{dx}(\cot x) = -\csc^2 x$

INVERSE TRIGONOMETRIC FUNCTIONS

19. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1 - x^2}}$

20. $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1 - x^2}}$

21. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1 + x^2}$

22. $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2 - 1}}$

23. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2 - 1}}$

24. $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1 + x^2}$

HYPERBOLIC FUNCTIONS

25. $\frac{d}{dx}(\sinh x) = \cosh x$

26. $\frac{d}{dx}(\cosh x) = \sinh x$

27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

28. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

29. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

30. $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

INVERSE HYPERBOLIC FUNCTIONS

31. $\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1 + x^2}}$

32. $\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2 - 1}}$

33. $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1 - x^2}$

34. $\frac{d}{dx}(\operatorname{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2 + 1}}$

35. $\frac{d}{dx}(\operatorname{sech}^{-1}x) = -\frac{1}{x\sqrt{1 - x^2}}$

36. $\frac{d}{dx}(\coth^{-1}x) = \frac{1}{1 - x^2}$