Exam #1
EGR 1010 Introductory Mathematics for Engineering Applications
Fall, 2014

Instructions: This exam consists of 5 problems worth a total of 100 points. The only materials permitted are a calculator and both sides of an 8.5"x11" HANDWRITTEN crib sheet, which must be turned in with the exam. Be sure to show all your work, and to include physical units on each final answer. POINTS WILL BE DEDUCTED FOR MISSING UNITS.

1. The number of college courses that Janice has completed toward her engineering degree after her 1st and 3rd year of college is summarized in the table below:

t, (years)	$C_{total}(t)$, (# of courses)
1	10
3	26

Assume the number of college courses that Janice has completed satisfies the following linear relationship:

$$C_{total}(t) = mt + C_{HS}$$

where m is the number of college courses completed per year and C_{HS} is the number of college courses she completed while still in high school.

a) Find the equation of the line $C_{total}(t)$, and determine both m and C_{HS}. (10 points)

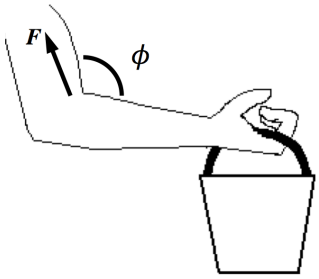
b) Sketch the graph of the line $C_{total}(t)$, and clearly indicate m and C_{HS} on the graph. (5 points)

c) If it takes 40 total courses for Janice to complete her engineering degree and she keeps taking courses at the same annual rate, how long will it take for Janice to finish her degree? (5 points)

Total Points: 20

2. A bicep muscle can apply a force F measured in Newtons (N) as a function of the elbow angle ϕ measured in degrees as described by the following quadratic equation:

$$F(\phi) = -0.04\phi^2 + 6\phi$$

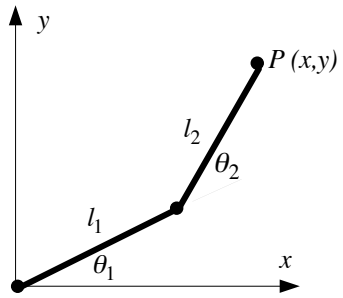


a) For a bicep force of F = 200 N, solve the equation for ϕ by each of the following methods; factoring, completing the square, and the quadratic formula. (15 points)

b) Using your solution from part a), determine the elbow angle ϕ where the force exerted by the bicep is maximum. In addition, calculate the maximum force F_{max}. (5 points)

Total Points: 20

3. Consider a two-link planar robot, with positive orientations of θ_1 and θ_2 as shown in the figure below:

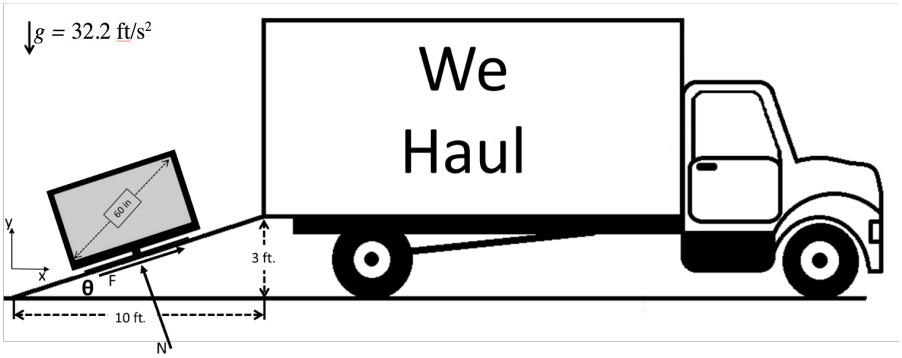


a) Suppose $\theta_1 = -30^\circ$, $\theta_2 = 120^\circ$, $l_1 = 5$ in and $l_2 = 3$ in. Sketch the orientation of the robot in the *x*-*y* plane, and determine the *x*-*y* coordinates of point *P*. (10 points)

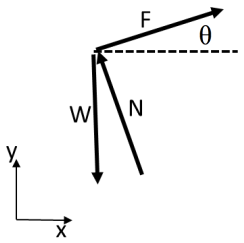
b) Suppose now that the same robot (i.e. $l_1 = 5$ in and $l_2 = 3$ in) is located in the *fourth* quadrant and is oriented in the "elbow up" position (i.e., with a positive value of θ_2). If the tip of the robot is located point $P(x,y) = (6,-4)$, determine the values of θ_1 and θ_2 using the Laws of Sines and Cosines. (10 points)

Total Points: 20

4. A 60-inch television, with a *mass* of 3.11 slugs, is loaded onto a truck using a ramp. A vector diagram showing the corresponding balance of forces acting on the TV is also given.



Vector Diagram:



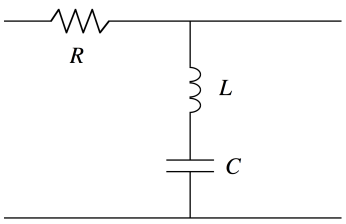
a) Based on the dimensions shown in the figure, determine the angle θ. (4 points)

b) With the positive *x*-y coordinate system given, express the normal force $\vec{N}$, the frictional force $\vec{F}$, and the weight $\vec{W}$ in standard vector notation (i.e. in terms of unit vectors $\vec{\imath}$ and $\vec{\jmath}$). (6 points)
NOTE: The weight W in pounds (lbf) is given by W = mg, where g=32.2 ft/s² is the acceleration due to gravity and m is the mass in slugs.

c) Substitute your results from part b) into the equilibrium equation $\vec{F} + \vec{N} + \vec{W} = 0$ and determine the values *F* and *N* in lbf. (10 points)

Total Points: 20

5. The circuit shown below consists of a resistor R, an inductor L, and a capacitor C. The impedance of the resistor is $Z_1 = R$, the impedance of the inductor is $Z_2 = j\omega L$, and the impedance of the capacitor is $Z_3 = 1/(j\omega C)$, where $j = \sqrt{-1}$.



For a low-pass crossover design, $R = 25\ \Omega$, $L = 25$ mH (x10^{-3} H), $C = 5\ \mu F$ (x10^{-6} F), and $\omega = 1000\pi$ rad/s.

a) Express the impedances Z_1, Z_2, and Z_3 in both their rectangular and polar forms. (5 points)

b) Compute the transfer function H of the crossover as given below and express the result in both rectangular and polar forms:

$$H = \frac{Z_2 + Z_3}{Z_1 + Z_2 + Z_3}$$

(10 points)

c) Determine the complex conjugate H^* and compute the product $H \cdot H^*$. (5 points)

Total Points: 20