## Exam \#2 <br> EGR 1010 Introductory Mathematics for Engineering Applications

Fall, 2014
Instructions: This exam consists of 5 problems worth a total of 100 points. The only materials permitted are a calculator and both sides of an 8.5 "x11" HANDWRITTEN crib sheet, which must be turned in with the exam. Be sure to show all your work and to include physical units on each final answer. POINTS WILL BE DEDUCTED FOR MISSING UNITS.

1. Consider the $R C$ circuit shown below, where the currents in amps are $i_{1}(t)=7 \sin (\pi t)$ and $i_{2}(t)=4 \sqrt{2} \sin (\pi t+\pi / 2)$ :

a) Given that $i(t)=i_{1}(t)+i_{2}(t)$, write $i(t)$ in the form $i(t)=M \sin (\pi t+\phi)$ (i.e., find $M$ and $\phi$ ). (10 points)
b) Suppose now that $i(t)=9 \sin \left(\pi t+38.9^{\circ}\right)$ Amps. Write down the amplitude, frequency (in Hertz), period, phase angle (in radians), and time shift (in seconds) of the current $i(t)$. ( 5 points)
c) Given your results of part b), plot one cycle of the function $i(t)=9 \sin \left(\pi t+38.9^{\circ}\right)$, and clearly indicate the earliest time when it reaches its maximum value. ( 5 points)

Total Points: 20
2. The following is the free body diagram of the arm showing the vertical and horizontal components of the force exerted by the deltoid muscle measured in Newtons (N).


The horizontal $\mathrm{F}_{\mathrm{H}}$ and vertical $\mathrm{F}_{\mathrm{V}}$ components of the deltoid muscle force for the configuration shown above satisfy the following system of equations:

$$
\begin{gathered}
\mathrm{F}_{\mathrm{V}}=0.22 \mathrm{~F}_{\mathrm{H}} \\
10-0.10 \mathrm{~F}_{\mathrm{V}}+0.01 \mathrm{~F}_{\mathrm{H}}=0
\end{gathered}
$$

a) Find $\mathrm{F}_{\mathrm{V}}$ and $\mathrm{F}_{\mathrm{H}}$ using the substitution method. (5 points)
b) Write the system of equations in matrix form $\mathbf{A F}=\mathbf{B}$, where $\mathbf{F}=\left[\begin{array}{l}F_{V} \\ F_{H}\end{array}\right]$. (5 points)
c) Find $\mathrm{F}_{\mathrm{V}}$ and $\mathrm{F}_{\mathrm{H}}$ using Cramer's Rule. Show all steps. (5 points)
d) Find $F_{V}$ and $F_{H}$ using the matrix algebra method. Perform all matrix computations by hand and show all steps. (5 points)
Total Points: 20
3. The compressive stresses under the discharge vane in a scroll compressor are observed to be $\sigma_{c}(t)=9.65-2.95 \cos (120 \pi t)$, where $t$ is measured in seconds and $\sigma_{c}(t)$ in ksi:

a) Determine the first three times where the slope of $\sigma_{c}(t)$ is zero. (i.e., $d \sigma_{c} / d t=0$ ). (5 points)
b) Determine if the compressive stresses at those times are maximums or minimums by using $d^{2} \sigma_{c} / d t^{2}$. (5 points)
c) Determine the compressive stresses $\sigma_{c}(t)$ at the times from part a). (5 points)
d) Using your results, sketch the compressive stress just under the discharge vane, $\sigma_{c}(t)$ for $0 \leq t \leq 1 / 60$ seconds. Clearly indicate the location and magnitude of the maximum compressive stress. (5 points)
4.
i) The acceleration of a vehicle starting from rest and at position $x=0$ (i.e., $v(0)=0 \mathrm{~m} / \mathrm{s}$ and $x(0)$ $=0 \mathrm{~m}$ ) is as shown in the figure below:

a) Knowing that $a(t)=d v / d t$, sketch the velocity $v(t)$. (5 points)
b) Knowing that $v(t)=d x / d t$, sketch the position $x(t)$ if the maximum position and the final position is $x(8)=100 \mathrm{~m}$. ( 5 points)
ii) A current is applied to a capacitor that has a capacitance $C=200 \mu F\left(200 \times 10^{-6} F\right)$ as shown in the figure below:

a) Knowing that $i(t)=C \frac{d v(t)}{d t}$, sketch the voltage across the capacitor $v(\mathrm{t})$. Note that the time is measured in $m s$ and the initial voltage is zero (i.e., $v(0)=0)$. ( 5 points)
b) Sketch the power if $p(\mathrm{t})=i(\mathrm{t}) v(\mathrm{t})$. (5 points)

Total Points: 20
5. A cantilever beam is subject to a distributed load $w$ and a moment $M$ at $x=L$.


If the value of the moment is $M=7 w L^{2} / 24$, then the deflection $y(x)$ is given by

$$
y(x)=\frac{w x^{2}}{48 E I}\left(8 L x-2 x^{2}-5 L^{2}\right)
$$

a) Determine the slope $\theta(x)=d y / d x$. ( 5 points)
b) Determine both the location and value of the maximum deflection. (5 points)
c) Evaluate both the deflection and slope at the points $x=0$ and $x=L$. (5 points)
d) Use your results from parts $\mathbf{b}$ ) and c ) to sketch the deflection $y(x)$. (5 points)

Total Points: 20

