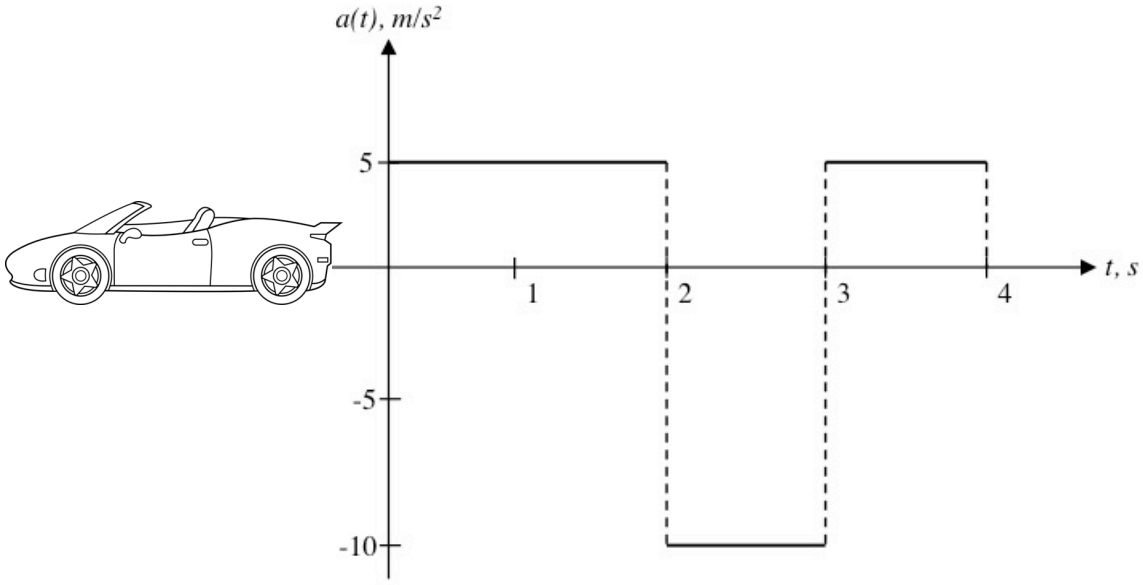
Final Exam
EGR 1010 Introductory Mathematics for Engineering Applications
Fall, 2014

Instructions: This exam consists of 5 problems worth a total of 100 points. The only materials permitted are a calculator and a total of three (3) 8.5"x11" HANDWRITTEN crib sheets, which must be turned in with the exam. Be sure to show all your work, and to include physical units where appropriate. POINTS WILL BE DEDUCTED FOR MISSING UNITS.

1. The acceleration profile of a vehicle is shown below:

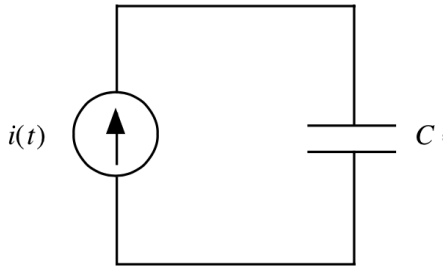


a) Use your knowledge of derivatives and/or integrals to plot the velocity $v(t)$. Assume the initial velocity is $v(0) = 0$ m/s. Clearly indicate the *maximum* and *minimum* velocity on your graph. (10 points)

b) Given your results of part a), use your knowledge of derivatives and/or integrals to plot the position $x(t)$, and clearly indicate the *maximum* value and *final* value of the position on your graph. Assume the initial position is $x(0) = 0$ m. (10 points)

Total Points: 20

2. A current $i(t) = 50e^{(-5t)}$ mA is applied to a capacitor of C=1000 μF, as shown in the figure below:



a) Knowing that $i(t) = C\ dv/dt$, integrate both sides of the equation to determine the voltage $v(t)$. The initial voltage is **$v(0)$=50 volts**. (5 points)

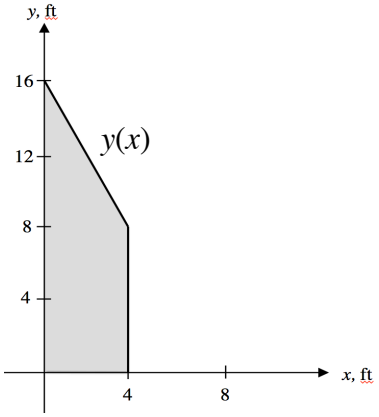
b) Evaluate the voltage $v(t)$ for t = 0.25 sec, t = 0.50 sec, and t = 0.75 sec and use your results to plot $v(t)$ for $0 < t < 1$ sec. (5 points)

c) Suppose the voltage through the capacitor is $v(t) = 10(6 - e^{-5t})$ volts. Compute the power $p(t) = v(t)i(t)$. (5 points)

d) Suppose that the power $p(t) = 3e^{-5t} - 0.5e^{-10t}\ W$. Knowing that $p(t) = dW/dt$, integrate both sides of the equation and calculate the stored energy assuming the initial energy is zero (i.e., W(0)=0 J). (5 points)

Total Points: 20

3. The tailfin of cruise missile has the cross sectional area as shown below:



a) Determine the equation of the line $y(x)$. (5 points)

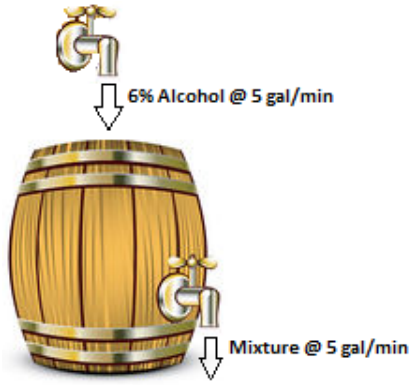
b) Determine the area of the tailfin by integration with respect to x. (5 points)

c) Determine the x-coordinate of the centroid by integration with respect to x. (5 points)

d) Determine the y-coordinate of the centroid by integration with respect to x. (5 points)

Total Points: 20

4. During the production process at a local brewery, a batch of beer with 6% alcohol is pumped into a barrel containing a 500 gallon batch of beer with 4% alcohol at a rate of 5 gallons/minute. The resulting mixture is pumped out at the same rate.



As the two batches mix, the total volume of pure alcohol in the barrel $a(t)$ changes as a function of time and satisfies the following first-order differential equation and initial condition:

$$\dot{a} + 0.01a(t) = 0.3,$$

$$a(0) = 20 \text{ gallons of alcohol}$$

a) Determine the transient solution, $a_{trans}(t)$. (5 points)

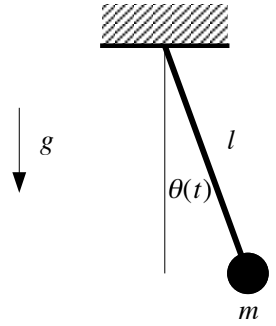
b) Determine the steady-state solution, $a_{ss}(t)$. (5 points)

c) Determine the total solution $a(t)$, subject to the given initial condition. (5 points)

d) Plot $a(t)$ as a function of time for $0 < t < 240$ min, and determine the percentage of alcohol in the barrel after 1 hour (60 min). (5 points) HINT: Percentage of alcohol = $a(60)/500$.

Total Points: 20

5. A simple pendulum of length l and mass m oscillates in the vertical plane, as shown in the figure below:



If the pendulum is initially displaced by a small angle θ_o, the oscillation $\theta(t)$ satisfies the following second-order differential equation and initial conditions:

$$ml\ddot{\theta} + mg\theta(t) = 0$$
$$\theta(0) = \theta_o,\ \dot{\theta}(0) = 0$$

where g is the acceleration due to gravity.

a) Determine the total solution for $\theta(t)$, subject to the initial conditions. (10 points)

b) Plot one cycle of the oscillation $\theta(t)$. Clearly label both its maximum and minimum values and the time it takes to get there. (5 points)

c) Mark each of the following statements as true (T) or false (F). (1 point each)

____ Increasing the length l will increase the amplitude of the oscillation.

____ Decreasing the length l will increase the frequency of the oscillation.

____ Increasing the mass m will increase the amplitude of the oscillation.

____ Decreasing the mass m will increase the frequency of the oscillation.

____ Increasing the initial displacement θ_o will increase the amplitude of the oscillation.

Total Points: 20