

Laboratory 2

Application of Trigonometry in Engineering

2.1 Laboratory Objective

The objective of this laboratory is to learn basic trigonometric functions, conversion from rectangular to polar form, and vice-versa.

2.2 Educational Objectives

After performing this experiment, students should be able to:

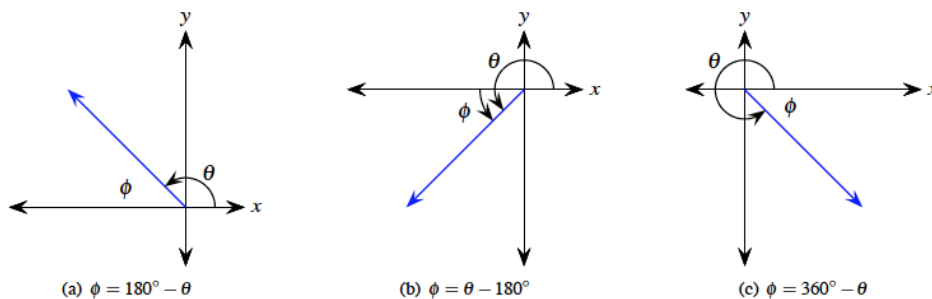
1. Understand the basic trigonometric functions.
2. Understand the concept of a unit circle and four quadrants.
3. Understand the concept of a reference angle.
4. Be able to perform the polar to rectangular and rectangular to polar coordinate conversion.
5. Prove a few of the basic trigonometric identities.

2.3 Background

Trigonometry is a tool that mathematically forms geometrical relationships. The understanding and application of these relationships are vital for all engineering disciplines. Relevant applications include automotive, aerospace, robotics, and building design. This lab will outline a few common, but useful, trigonometric relationships.

2.3.1 Reference Angle

A reference angle is an acute angle (less than 90°) that may be used to compute the trigonometric functions of the corresponding obtuse angle (greater than 90°). Figure 2.1 shows the reference angle ϕ with respect to the angle θ . The reference angle is calculated using the formulas shown in the captions of each corresponding subfigure of Figure 2.1



2.3.2 Law of Cosines

The Law of cosines is a method that helps to solve triangles. Equations 2.1 relate the sides and interior angles of Figure 2.2.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos(A) \\ b^2 &= a^2 + c^2 - 2ac \cos(B) \\ c^2 &= a^2 + b^2 - 2ab \cos(C) \end{aligned} \tag{2.1}$$

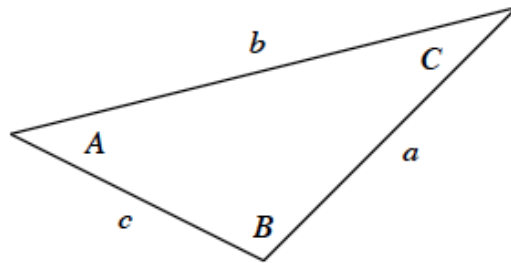


Figure 2.1: Law of Cosines Triangle

2.3.3 Law of Sines

The Law of Sines is another method that helps to solve triangles. Using the triangle of Figure 2.1, Equation 2.2 relates the sides to the interior angles.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \tag{2.2}$$

2.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the laboratory equipment.

2.4.1 One Link Robot

1. Using the boards in the lab, fill in Table 2.1. Pay close attention to the sign of your answer for all values.

NOTE: To convert a value in degrees to radians, the multiplying factor is $\pi/180$.

2. Use equation 2.3 to find the calculated x and y values.

$$x = l \cos(\theta) \tag{2.3}$$

$$y = l \sin(\theta)$$

3. Using the boards in the lab, fill in Table 2.2.
4. Use Equations 2.4 to find the Calculated θ and l .

$$\theta = \tan^{-1}(y/x)$$

$$l = \sqrt{x^2 + y^2}$$
(2.4)

2.4.2 Trigonometric Identities

An identity is a trigonometric relationship that is true for all permissible values of the variable(s). Many times, trigonometric identities are used to simplify more complex problems.

1. Using MATLAB, fill in Tables 2.3 and 2.4.
 - a. The first column of Table 2.3 comes from 5th column of Table 2.2.
 - b. Define this column as a vector in MATLAB and perform element by element calculations on it to get the other columns.

NOTE: All calculations should be done with MATLAB. No calculator use!

2.4.3 Two Link Robot

1. Using the boards in the lab, fill in the Measured Values of Table 2.5.
NOTE: $l_1 = l_2 = 50$ mm
2. Write a MATLAB code to calculate x and y by adding the components of each link. Recall the following equations from class.

$$x_1 = l_1 \cos(\theta_1)$$

$$y_1 = l_1 \sin(\theta_1)$$

$$x_2 = l_2 \cos(\theta_1 + \theta_2)$$

$$y_2 = l_2 \sin(\theta_1 + \theta_2)$$

$$X = x_1 + x_2$$

$$Y = y_1 + y_2$$

2.4.4 Solve a Triangle Using Law of Cosines and Law of Sines

In some cases, the laws of sines and cosines must both be used to solve a triangle. Figure 2.3 is one such case where the lengths l_1 and l_2 along with the final ending point P of the two links are known and the \square values are not. Both laws are needed to solve this triangle.

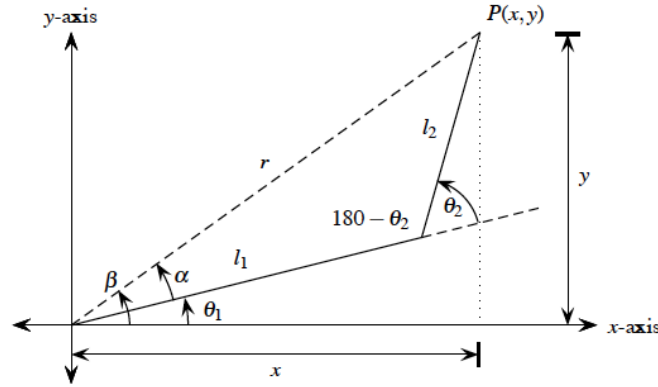


Figure 1.3: General Two Link Robot

1. The radius r is found by:

$$r = \sqrt{x^2 + y^2}$$

2. Using the Law of Cosines, θ_2 is found by the following equation:

$$r^2 = l_1^2 + l_2^2 - 2l_1l_2\cos(180 - \theta_2)$$

$$\theta_2 = 180 - \cos^{-1} \frac{r^2 - l_1^2 - l_2^2}{-2l_1l_2} \quad (2.6)$$

3. Using the Law of Sines, α is found by the following equation:

$$\frac{\sin(\theta_2)}{r} = \frac{\sin(\alpha)}{l_2}$$

$$\alpha = \sin^{-1} \left(\frac{l_2 \sin(\theta_2)}{r} \right) \quad (2.7)$$

4. θ_1 is now found by using the equations:

$$\beta = \tan^{-1} \left(\frac{y}{x} \right) \quad (2.8)$$

$$\theta_1 = \beta - \alpha \quad (2.9)$$

2.5 Lab Requirements

1. Write an abstract for this lab and submit it to the Lab 2 folder in your lab section's abstract folder found in the Pilot Dropbox. (Required to pass course.)
2. Complete Tables 2.1 – 2.6. (2 points each)

Table 2.1 Polar to Rectangular Conversion

Angle θ°	Measured x (mm)	Measured y (mm)	Vector Form $x\hat{i} + y\hat{j}$	l (mm)	Reference Angle ($^\circ$)	Reference Angle (rad)	Calculated x (mm)	Calculated y (mm)
30				100				
45				100				
90				100				
135				100				
180				100				
225				100				
270				100				

Table 2.2 Rectangular to Polar Conversion

(x,y)	Measured $\theta(^\circ)$	Reference Angle ($^\circ$)	Reference Angle (rad)	Calculated $\theta(^\circ)$	Calculated l (mm)	Polar Form $l\angle\theta$
(85,50)						
(70,70)						
(0,100)						
(-70,70)						
(-100,0)						
(-70,-70)						

Table 2.3

Calculated $\theta(^{\circ})$	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\sec(\theta)$

Table 2.4

$\frac{\sin(\theta)}{\cos(\theta)}$	$\sin^2(\theta) + \cos^2(\theta)$	$1 + \tan^2(\theta)$	$\sec^2(\theta)$

Table 2.5

		Measured Values						Calculated Values	
$\theta_1(^{\circ})$	$\theta_2(^{\circ})$	x_1	y_1	x_2	y_2	$X = x_1 + x_2$	$Y = y_1 + y_2$	X	Y
0	0								
0	90								
30	45								
30	60								
180	0								
270	30								
360	90								

Table 2.6 Application of Sine and Cosine Laws

$P(x,y)$ mm	θ_2 (°)	α (°)	β (°)	θ_1 (°)
(55, 75)				
(75, 60)				
(15, 63)				
(32, 14)				
(71, 70)				

3. Publish your MATLAB for Tables 2.3 through 2.6. (2 points each)
4. Answer the following questions.
 - a) Based on your results for Tables 2.3 and 2.4, write down the three trigonometric identities that were verified. (2 points)