# Laboratory 4

# Systems of Equations in Engineering: The Two-Loop Circuit

## 4.1 Laboratory Objective

The objective of this laboratory is to learn the basics of systems of equations and matrices and their application in engineering.

## 4.2 Educational Objective

After performing this experiment, students should be able to:

1.  Solve for the unknowns by use of a matrix inverse, Cramer's Rule, substitution, and MATLAB.

## 4.3 Background

Simultaneous equation solving is a key skill in many engineering applications. For example, software in finite element modeling and thermodynamics solve multiple equations with multiple variables. MATLAB is one such piece of software that can solve simultaneous equations. The following methods will solve relatively small problems easily and can provide an answer quickly, however with larger systems, hand calculations can be exhaustive and software becomes the more intelligent route to the solution.

### 4.3.1 Problem Statement

A system of equations can be written as $Ax = b$, where $A$ is the coefficient matrix, $x$ is a vector of unknowns and $b$ is a vector of the right hand sides of the equations. For illustration, the following matrices will be used for explaining the methods.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

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$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

## 4.3.2 Matrix Inverse Method

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

## 4.3.3 Cramer's Rule

$$x_1 = \frac{\begin{vmatrix} b_1 & b \\ b_2 & d \end{vmatrix}}{ad - bc}$$

$$x_2 = \frac{\begin{vmatrix} a & b_1 \\ c & b_2 \end{vmatrix}}{ad - bc}$$

## 4.3.4 Substitution

The system $A\vec{x} = \vec{b}$ can be written as two equations:

$$ax_1 + bx_2 = b_1$$

$$cx_1 + dx_2 = b_2$$

Solve for $x_2$ in Equation 4.3 and substitute into Equation 4.2.

$$a\left(\frac{b_2 - dx_2}{c}\right) + bx_2 = b_1$$

$$x_1 = \left(\frac{b_1 - \frac{ab_2}{c}}{b - \frac{ad}{c}}\right)$$

Once $x_2$ is known, substitute back into either Equation 4.2 or 4.3, and solve for $x_1$.

**4.3.5 MATLAB**

$$\vec{x} = A^{-1}\vec{b}$$

$$\vec{x} = A\backslash\vec{b}$$

NOTE: The second method is pronounced "A left division b" and is much more efficient for larger matrices than method one.

## 4.4 Procedure

1. The currents flowing through loop 1 and loop 2 in the circuit below are unknown. Construct the circuit and use the lab equipment to find these currents. Complete Tab le 4.1.

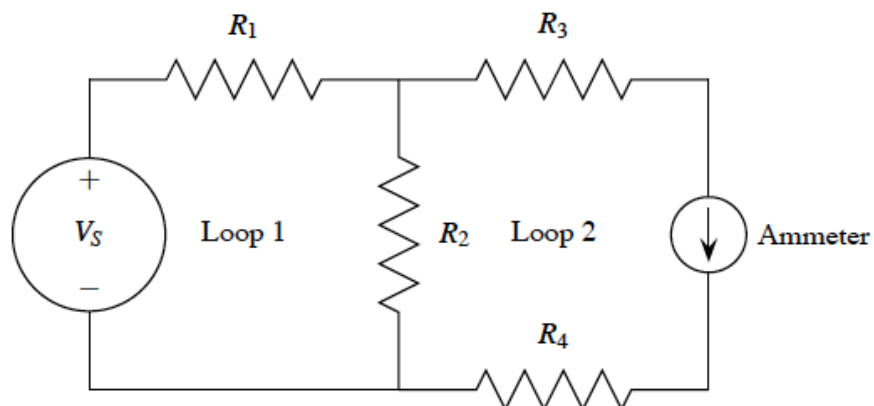   NOTE: $R_1 = R_2 = 100\Omega$ and $R_3 = R_4 = 200\Omega$.



Figure 4.1: A Two-Loop Circuit

## 4.5 Lab Requirements

1. Write an abstract for this lab and submit it to the Lab 4 folder in your lab section's abstract folder found in the Pilot Dropbox. (Required to pass course.)

2. Complete Table 4.1. (2 points)

**Table 4.1** Two-Loop Circuit

| $V_S$ (Volts) | $I_1$ (Amps) | $I_2$ (Amps) |
|---|---|---|
| 5 | | |
| 7 | | |

3. By hand, calculate the unknown currents, $I_1$ and $I_2$ using the three methods below and attach after this sheet. (2 points each)

   a) Inverse Matrix Method

   b) Cramer's Rule

   c) Substitution

   NOTE: Use the following matrix setup (Take $V_S$ to be 7):

$$\begin{bmatrix} (R_1 + R_2) & -R_2 \\ -R_2 & (R_2 + R_3 + R_4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \end{bmatrix}$$

4. Write a MATLAB code implementing the Inverse Matrix Method. Check your answer with "Left Division". Attach after this sheet: (4 points) (both cases where $V_S = 5$ and 7 V)

   a) m-file

   b) output

5. Compare your calculated values for $I_1$ and $I_2$ with the measured values. Why are they different? (2 points)