Grade:____/31

Laboratory 5

Derivatives in Engineering: Velocity and Acceleration in Free-Fall

5.1 Laboratory Objective

The objective of this laboratory is to illustrate the application of a derivative with a freefall exercise.

5.2 Educational Objective

After performing the experiment, students should be able to:

1. Understand the relationship between position, velocity, and acceleration.
2. Identify the key parameters of free-fall.
3. Use MATLAB symbolic function to calculate derivatives.

5.3 Background

The derivative is a tool that describes the rate of change of a quantity with respect to the change in another. Geometrically this is equivalent to slope.

5.3.1 Position, Velocity, and Acceleration

Given a function $y(t)$ that represents position with respect to time, one can derive the expressions for the velocity $v(t)$ and the acceleration $a(t)$. Velocity is simply the derivative of $y(t)$ with respect to time and acceleration is the second derivative of $y(t)$ with respect to time.

$$v(t) = \frac{dy(t)}{dt}$$

$$a(t) = \frac{d^2y(t)}{dt^2} = \frac{dv(t)}{dt}$$

Velocity can also calculated using:

$$v(t) = \frac{y_2 - y_1}{t_2 - t_1}$$

Similarly, acceleration can be calculated using:

$$a(t) = \frac{d^2y(t)}{dt^2} = \frac{dv(t)}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

The free-fall apparatus used in this lab consists of a free fall device, an ultrasonic sensor that is mounted to a fixed position, and a computer to record the data.

5.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the laboratory equipment.

1. Open Data Studio and click the Setup icon.
2. Set the sample rate to 20 Hz (ie. $\Delta t = 1/20sec$)
3. Close the setup window.
4. Start data collection and drop device.
5. Press stop after the object hits the ground.
6. Copy data into Microsoft Excel by dragging a box around the data and then copy/pasting.
7. Construct Table 5.1 in Excel and plot the measured position, velocity, and acceleration vs. time.

Table 5.1 Position, Velocity, and Acceleration

$y_i(m)$	$t_i(s)$	$\Delta y = (y_{i+1} - y_i)\ m$	$\Delta t = (t_{i+1} - t_i)\ s$	$v_i = \frac{\Delta y}{\Delta t}$ (m/s)	$\Delta v = v_{i+1} - v_i$ (m/s)	$a_i = \frac{\Delta v}{\Delta t}$ (m/s^2)
$y_0 = 0$	$t_0 = 0$	-	-	0	-	0
y_1	t_1	$y_1 - y_0$	$t_1 - t_0$	v_1	$v_1 - v_0$	a_1
y_2	t_2	$y_2 - y_1$	$t_2 - t_1$	v_2	$v_2 - v_1$	a_2
etc.	etc.	etc.	etc.	etc.	etc.	etc.

8. Repeat this procedure for 40 Hz and 50 Hz sample rates.

5.5 Lab Requirements

1. Write an abstract for this lab and submit it to the Lab 5 folder in your lab section's abstract folder found in the Pilot Dropbox. (Required to pass course.)

2. Insert all three Tables from Excel. (2 points each)

    a) 20 Hz

    b) 40 Hz

    c) 50 Hz

3. Plot all 3 frequencies for the measured position on one set of axes. (Don't forget to properly label the graphs!) (4 points)

4. Plot all 3 frequencies for the measured velocity on one set of axes. (4 points)

5. Plot all 3 frequencies for the measured acceleration on one set of axes. (4 points)

6. What is the average acceleration for each frequency? (3 points)

7. Write a MATLAB script that will plot (as a subplot) the theoretical (calculated) position, velocity, and acceleration vs. time. Use MATLAB subplot commands and publish your result. (2 points each)

NOTE: for free-fall, $y(t) = y_0 - v_0 t + \frac{1}{2} a t^2$

8. Answer the following questions:

    a) In free-fall, what physical quantity does the acceleration represent? (2 points)

    b) What is the mathematical relationship between position, velocity, and acceleration? (2 points)