

Laboratory 6

Integrals in Engineering: Work and Stored Energy in a Spring

6.1 Laboratory Objective

The objective of this laboratory is to illustrate the application of an integral by an exercise with spring work.

6.2 Educational Objective

After performing this experiment, students should be able to:

1. Understand that geometrically, an integral calculates area under a curve.
2. Understand the work done on a spring.

6.3 Background

Work is a fundamental concept of many physical systems. In general, the sum of all forces over a given distance is work.

$$W = \int_0^x F(x) dx \quad (6.1)$$

A spring has work done on it when it is stretched. The spring force is linearly related to the distance stretched by a constant k .

$$F(x) = kx \quad (6.2)$$

The work done on a spring by a mass can be found by substitution Equation 6.2 into Equation 6.1.

$$W = \int_0^x kx dx \quad (6.3)$$

Figure 6.1 below shows the setup that will be used in the lab.

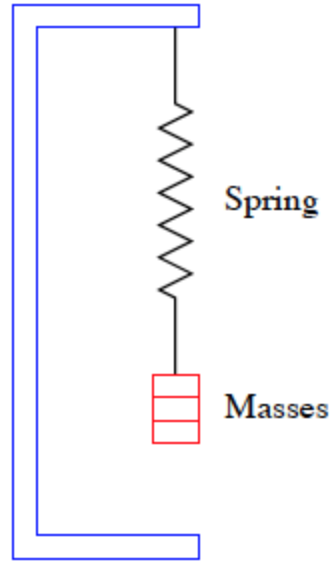


Figure 6.1: Spring & Mass System

6.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the laboratory equipment.

1. Attach the spring to the stand and suspend the mass hanger from the other end. Record the measurement from the scale. This is your reference value x_0 .
2. Complete Tables 6.1, 6.2, and 6.3.

a) Add mass as shown in each table and record the measurement y_i from the scale.

NOTE: The tick marks are measured in *centimeters (cm)*!

b) Calculate the force on the spring in Newtons. ($g = 9.81 \text{ m/s}^2$)

$$F = mg$$

c) Calculate the extension y_i due to each added mass.

NOTE: Remember to take the absolute value of your answers in column four if they are negative.

3. Plot F vs. y_i in MATLAB for all three Tables.
4. Using MATLAB's Basic Fitting tool, find the slope of the graphs.
5. Plot a bar graph of F vs. y_i in MATLAB for all three tables.

NOTE: MATLAB syntax is given in Section 6.6

6.5 Lab Requirements

1. Write an abstract for this lab and submit it to the Lab 6 folder in your lab section's abstract folder found in the Pilot Dropbox. (Required for passing the course).

2. Complete Tables 6.1, 6.2, and 6.3. (2 points each)

Table 6.1

Mass (Kg)	Scale y_i (m)	$F = mg$ (N)	$\Delta y = y_{i+1} - y_i $ (m)
0	$y_0 = 0$	0	-
0.08	$y_1 =$		
0.16	$y_2 =$		

Table 6.2

Mass (Kg)	Scale y_i (m)	$F = mg$ (N)	$\Delta y = y_{i+1} - y_i $ (m)
0	$y_0 = 0$	0	-
0.04	$y_1 =$		
0.08	$y_2 =$		
0.12	$y_3 =$		
0.16	$y_4 =$		

Table 6.3

Mass (Kg)	Scale y_i (m)	$F = mg$ (N)	$\Delta y = y_{i+1} - y_i $ (m)
0	$y_0 = 0$	0	-
0.02	$y_1 =$		
0.04	$y_2 =$		
0.06	$y_3 =$		
0.08	$y_4 =$		
0.10	$y_5 =$		
0.12	$y_6 =$		
0.14	$y_7 =$		
0.16	$y_8 =$		

3. Insert 3 linear plots after this page. (2 points each)

List the values of the slope for all 3: (3 points)

4. Insert 3 bar graphs after this page. (2 points each)

5. Write a MATLAB script that calculates the area of the bar graphs for all three tables and publish the results. (2 points)

NOTE: **bar(x,y,1)** Draws a bar graph with x values at the midpoint of each rectangle.

HINT: The widths of each bar should be column Δy from the tables.

6. Calculate the integral by hand. (2 points)

NOTE: Use the value of k that was found from Table 6.3 in the equation. The limits of integration are 0 to $|y_8|$ from Table 6.3.

$$W = \int_0^{y_8} kx \, dx$$

7. Answer the following questions:

a) What does the slope of the linear plots physically represent? (2 points)

b) What are the units of the spring constant k ? (2 points)

c) What do the areas of the bar graphs physically represent? (2 points)

d) How do the areas of the bar graphs compare to your answer for Question 6? (2 points)