Name:* ____** ***Grade:*___*/14

***Section Number:* _________**

Laboratory 8

Differential Equations in Engineering:

Spring-Mass Vibration

8.1 Laboratory Objective

The objective of this laboratory is to model spring-mass behavior with a second order differential equation.

8.2 Educational Objectives

After performing this experiment, students should be able to:

1. Apply principles of modeling and analysis to a spring-mass system.
2. Identify and measure the key parameters of a spring-mass system.
3. Validate a mathematical model (differential equation) with measured data.

8.3 Background

Another class of differential equations is second order applications. These equations contain a second derivative of the variable in question. In the case of a spring-mass system, the displacement as a function of time is the unknown quantity.

8.3.1 The Spring-Mass System

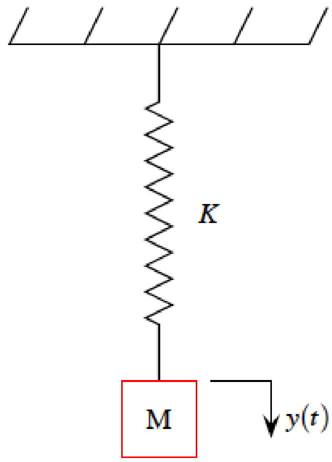

Figure 8.1: Spring-Mass System

The spring-mass system shown in Figure 8.1 has kinetic energy associated with the mass moving up and down and potential energy stored in the spring. This energy is passed back and forth as the spring oscillates. The free body diagram (FBD) in Figure 8.2 shows all forces acting on the mass.

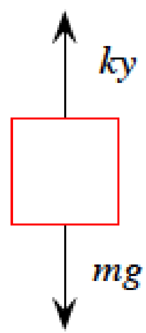

Figure 8.2: Free Body Diagram of Spring-Mass System

From equilibrium forces in the y-direction,

$$k\delta = mg,$$

Which gives

$$\delta = \frac{mg}{k}$$

This represents the static deflection of the spring. Once the mass is displaced from the equilibrium position and allowed to vibrate, the mass-spring system is no longer in equilibrium. Applying Newton's Second Law and simplifying:

$$\sum \mathrm{F} = ma$$

$$mg - k(\delta + y(t)) = m\ddot{y}(t)$$

$$mg - k\left(\frac{mg}{k}\right) - ky(t) = m\ddot{y}(t) \qquad (8.1)$$

$$m\ddot{y}(t) + ky(t) = 0 \qquad (8.2)$$

Equation 8.1 is the governing equation of a frictionless spring-mass system.
The solution to this equation is:

$$y_{total}(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

The mass will oscillate as a cosine wave with amplitude *A* and angular frequency $\sqrt{\frac{k}{m}}$.

8.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the equipment.

1. Attach the spring to the stand and suspend the mass hanger from the other end.
2. Complete Table 8.1 using a single spring and double springs in series with masses as indicated.

   NOTE: Use two identical springs for the double spring measurements.
3. Calculate the spring constants k_1 and k_2 with the following equation.

$$k_i = \left|\frac{g(m_1 - m_2)}{y_1 - y_2}\right|$$

4. Place the indicated mass on the hanger, displace it, and measure the time, *t* it takes to complete 20 cycles according to Table 8.2.
5. Calculate the period of oscillation $T_{measured}$ using Equation 8.3.

$$T_{measured} = \frac{t_{20}}{20} \qquad (8.3)$$

6. The theoretical $T_{calculated}$ can be found by Equation 8.4. The additional 0.2 kg is the mass of the hanger. Find the calculated period for all cases by completing Table 8.3.

$$T_{calculated} = 2\pi\sqrt{\frac{m+0.2}{k}} \qquad (8.4)$$

8.5 Lab Requirements

1. Write an abstract for this lab and submit it to the Lab 8 folder in your lab section's abstract folder found in the Pilot Dropbox. (Required to pass course.)

2. Complete Tables 8.1, 8.2, and 8.3. (2 points each)

Table 8.1

Single Spring		Double Spring	
Mass (kg)	y_i (m)	Mass (kg)	y_i (m)
0.08		0.08	
0.16		0.16	
k_1 =		k_2 =	

Table 8.2 – Measured Period

Single Spring		Double Spring		Single Spring		Double Spring	
m = 0.08kg				m = 0.16kg			
t_{20} (sec)	$T_{measured}$	t_{20} (sec)	$T_{measured}$	t_{20} (sec)	$T_{measured}$	t_{20} (sec)	$T_{measured}$

Table 8.3 – Theoretical Period

Single Spring	Double Spring	Single Spring	Double Spring
m = 0.08 kg		m = 0.16 kg	
T_{calc}	T_{calc}	T_{calc}	T_{calc}

3. Answer the following question:

   a. Compare $T_{measured}$ with T_{calc}. Why are they different? (2 points)