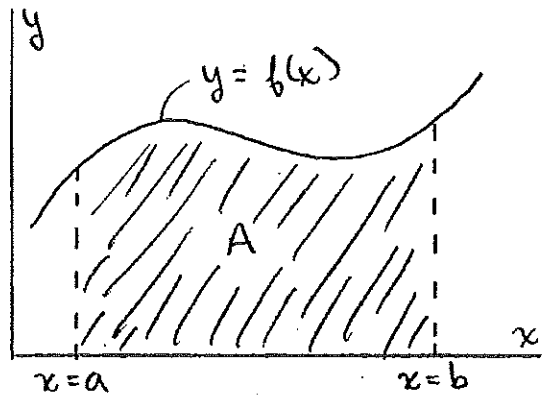
The Integral

The definite integral $\int_a^b f(x)dx$ is the area under the function $y = f(x)$ between points a and b:



$$A = \int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b) - F(a),$$

where $F(x) = \int f(x)dx$ is the *antiderivative* of $f(x)$.

Antiderivatives of some common functions in engineering:

Function, $f(x)$	Antiderivative, $F(x) = \int f(x)dx$
$\sin(\omega x)$	$-\frac{1}{\omega}\cos(\omega x) + C$
$\cos(\omega x)$	$\frac{1}{\omega}\sin(\omega x) + C$
e^{sx}	$\frac{1}{s}e^{sx} + C$
x^n	$\frac{x^{n+1}}{n+1} + C$
$cf(x)$	$c\int f(x)dx$
$f_1(x) + f_2(x)$	$\int f_1(x)dx + \int f_2(x)dx$

- In the above table, ω, s, n, c and C are constants (not functions of x)

TABLE OF INTEGRALS

EXPONENTIAL AND LOGARITHMIC FORMS

96. $\int ue^{au}\,du = \frac{1}{a^2}(au - 1)e^{au} + C$

97. $\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$

98. $\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2 + b^2}(a\sin bu - b\cos bu) + C$

99. $\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2 + b^2}(a\cos bu + b\sin bu) + C$

100. $\int \ln u\,du = u\ln u - u + C$

101. $\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

102. $\int \frac{1}{u\ln u}\,du = \ln|\ln u| + C$

HYPERBOLIC FORMS

103. $\int \sinh u\,du = \cosh u + C$

104. $\int \cosh u\,du = \sinh u + C$

105. $\int \tanh u\,du = \ln\cosh u + C$

106. $\int \coth u\,du = \ln|\sinh u| + C$

107. $\int \operatorname{sech} u\,du = \tan^{-1}|\sinh u| + C$

108. $\int \operatorname{csch} u\,du = \ln|\tanh\tfrac{1}{2}u| + C$

109. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

110. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

111. $\int \operatorname{sech} u\tanh u\,du = -\operatorname{sech} u + C$

112. $\int \operatorname{csch} u\coth u\,du = -\operatorname{csch} u + C$

FORMS INVOLVING $\sqrt{2au - u^2}$, $a > 0$

113. $\int \sqrt{2au - u^2}\,du = \frac{u-a}{2}\sqrt{2au - u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

114. $\int u\sqrt{2au - u^2}\,du = \frac{2u^2 - au - 3a^2}{6}\sqrt{2au - u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

115. $\int \frac{\sqrt{2au - u^2}}{u}\,du = \sqrt{2au - u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$

116. $\int \frac{\sqrt{2au - u^2}}{u^2}\,du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(\frac{a-u}{a}\right) + C$

117. $\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$

118. $\int \frac{u\,du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$

119. $\int \frac{u^2\,du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2}\sqrt{2au - u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

120. $\int \frac{du}{u\sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$